High speed beamforming using fast Fourier's transform and migration for arbitrary simultaneous plural transmissions and using virtual source behind transducer

Chikayoshi Sumi
Information and Communication Sciences
Faculty of Science and Technology
Sophia University
Tokyo Japan
c-sumi@sophia.ac.jp

ITEC Sep 21-24 2015, Lake Garda (ID, 039)

Submission on Sep 21, 2015.

Correction of a typographical error in page 12 (Nov 23, 2015).

Background and purpose

A high frame rate is required to achieve high accuracy measurement of rapid tissue motion or shear wave propagation, or 3D imaging, which are all important for elastography or other elasticity imaging. It is also required to generate lateral frequencies, i.e., via lateral modulation. For such purposes, we have been evaluating the simultaneous transmissions of plural, steered plane waves, and cylindrically or spherically focused beams (e.g., paper ID, 040). That is, the plural waves or beams are transmitted at the same time. For these transmissions, reception beamforming must be performed with a considerably high speed and accuracy. Thus far, only for a single plane wave transmission, several Fourier based beamforming methods were proposed [1-3].

[1] J. Cheng, J.-y. Lu, IEEE Trans UFFC, vol. 53, pp. 880-899, 2006. [2] P. Kruizinga et al, IEEE Trans. UFFC, vol. 59, pp. 2684-2691, 2012. [3] D. Garcia, et al, IEEE Trans. UFFC, vol. 60, pp. 1853-1867, 2013.

We have been reporting new wavenumber matching methods used in Fourier's transform-based beamforming methods. All other groups performed wavenumber matching approximately by interpolating spectra in a frequency domain or using a special Fourier's transform with nonuniform sampling intervals

In contrast, our new wavenumber matching methods permit to carry out arbitrary beamforming on an arbitrary coordinate system without any approximate calculations. For such echo data generation, a high accuracy and a high speed are achieved. For instance, the single plane wave beamforming was reported at this conference in 2013.

Accordingly, the accuracy in a displacement measurement is also improved.

cf. Recall that L. J. Busse performed non-approximate wave matching for monostatic synthetic aperture (SA), however, with no steering. L. J. Busse, IEEE Trans. UFFC, vol. 39, 174, 1992.

We have achieved arbitrary beamforming with Fourier's transform and no approximate wavenumber matching.

- ☐ Multiple focusing and plane wave transmissions
- ☐ Achieved on an arbitrary coordinate system, i.e., different coordinate systems for physical transmission and reception signal processing, e.g., radial scan (convex, sector and IVUS etc.) can be performed on Cartesin coordinate system.

(1) Plane wave transmission with steering

(2) Monostatic synthetic aperture (SA) with steering

(1)+(2) Combination of (1) and (2) [an effective beamforming, C. Sumi, IEEE Trans UFFC (2008)]

(3) Multistatic SA with steering

(4) Fixed focusing with steering

(5) Above-listed beamforming on an arbitrary coordinate system including a polar coordinate system

(6) Simultaneous signal processing for simultaneous transmissions

(7) Virtual source set behind physical aperture

(8) Above-listed beamforming using migration method

(1) Plane wave transmission with steering

J.-y. Lu's method

1. FFT on time, t
2. FFT on lateral axis, x
3. Matching on (kx,ky) via interpolation
4. 2D IFFT on (x,y)

The use of new method

1. FFT on time, t
2. Matching on kx $(\times e^{ik\sin\theta x})$
3. FFT on lateral axis, x
4. Matching on ky $(\times e^{ik\cos\theta y})$ and calculation of angular spectra at respective depth y $(\times \exp(i\sqrt{k^2-(k_x-k\sin\theta)^2}y))$, i.e., simultaneous multiplication
5. Summation with respect to k
6. IFFT on x

Our method achieves no approximation and with respect to IFFT, only 1D IFFT is performed.

Combination of steering

About Method (1):

Physical transmission steering α;

Transmission steering angle θ via signal processing (software)

Steering angle can be generated as a mean of physical and software transmission steering angle (α+θ).

For instance, if α=θ, generated steering angle becomes α(=θ).

About Method (1)+(2), i.e., plane wave transmission and dynamic focusing reception:

Physical transmission steering α;

Transmission (β) and reception (γ) steering angle via signal processing (software)

Steering angle can be generated as a mean of total transmission steering angle (α+β) and reception steering angle γ.

- In a mathematical sense, the software transmission and reception beamforming can be interpreted exchangeable.

  This is also for other beamforming including Method (8).

- Arbitrary transmission beamforming can be dealt with using Method (1) or Method (1)+(2).

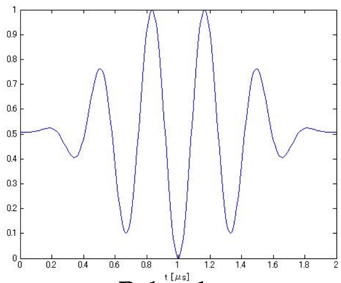
  e.g., steered plane wave, steered focused beam, synthetic aperture with steering (dynamic focusing), non-steered beam or wave etc.

- Steering angle can be increased or decreased via signal processing.

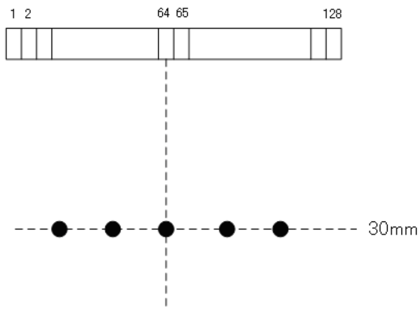
Simulations using Field II

- 1D linear type array probe:
  element width, 0.1 mm;
  kerf, 0.025 mm;
  elevation width, 5mm:
  128 elements;
  US freq., 3MHz.

- 5 point scatters (2.5 mm intervals) situated at depth 30 mm in anechoic and no attenuation medium.

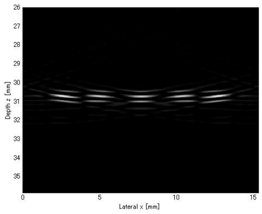
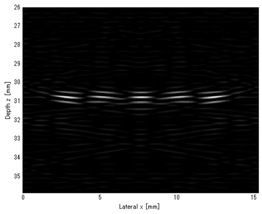
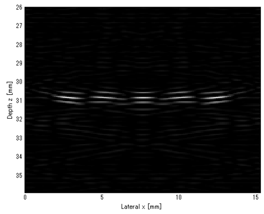


Pulse shape



Field II: J. A. Jensen, Med Biol Eng Comp, 10th Nordic-Baltic Conf on Biomed Imag, vol. 4, 1996

Differences in non-approximate and approximate wavenumber matching, e.g., steering angle =0°

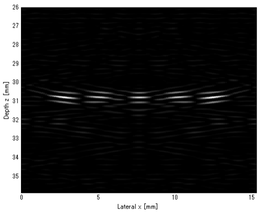
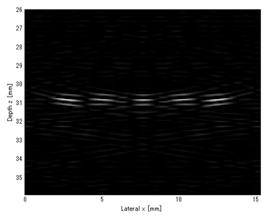
No approximation | fs=100MHz | fs=25MHz

Sampling freq. fs=100MHz

=>

Down sampling in a freq. domain to 10MHz; and echo data generated with 25MHz (step 4).

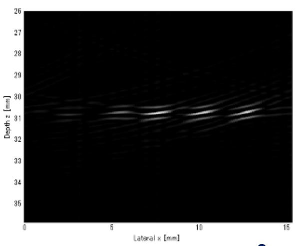
Approximation using neighborhood spectra

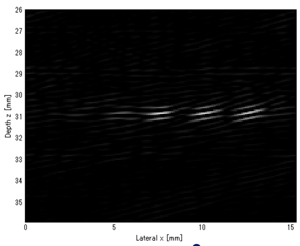



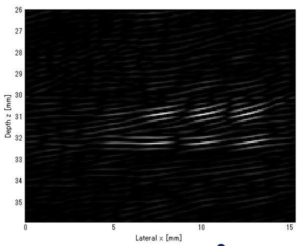
fs=100MHz | fs=25MHz

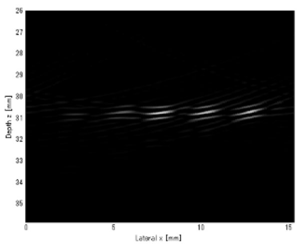
Linear interpolation

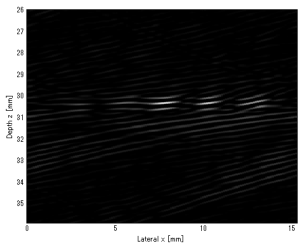
Interpolation only for axial direction (’) and both directions (”) using neighborhood spectra. Steering angle, 10° . (Upper) Method (1) and (lower) Method (8).

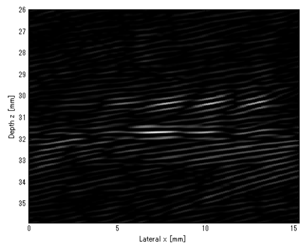

Method (1) 10.3°


(1’) 9.3°


(1”) 10.5°

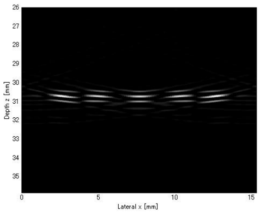

Method (8) 9.1°


(8’) 6.9°

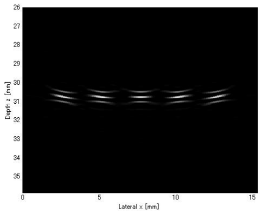

(8”) 8.5°

Increasing in lateral spatial resolution via coherent compounding (1° digit)

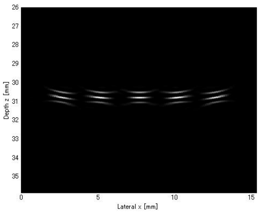
Angular spectra are compounded in a frequency domain.



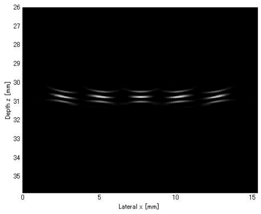
1 wave(0°)



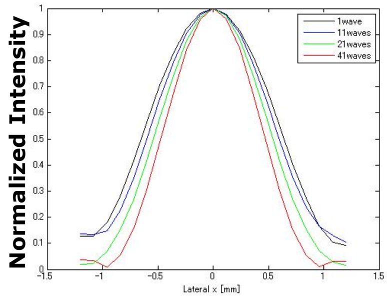
11 waves (-5° ~5°)



21 waves (-10° ~10°)



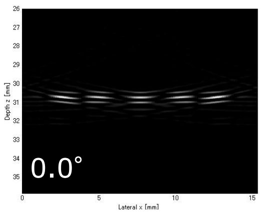
41 waves (-20° ~20°)

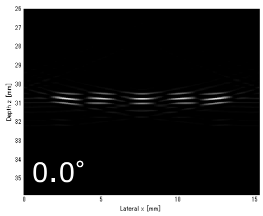


Lateral beam profile

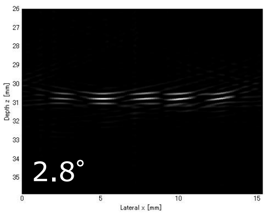
Comparison

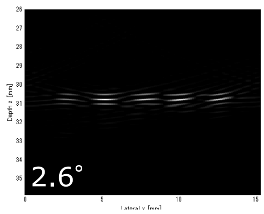
(a) θ = 0°



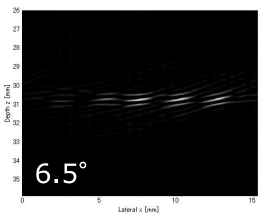


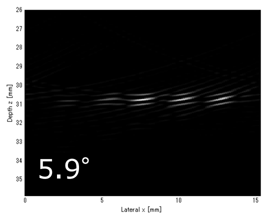
(b) θ = 5°





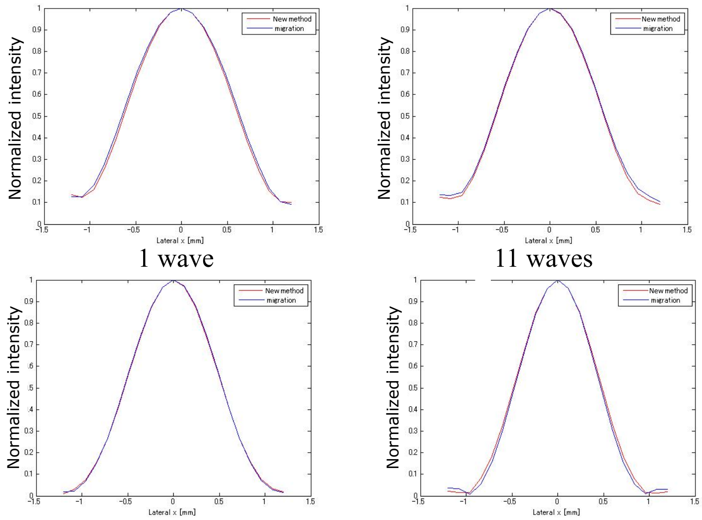
(c) θ = 10°





Method (1) Method (8)

Comparison on coherent compounding



1 wave — 11 waves — 21 waves — 41 waves

Lateral beam profiles

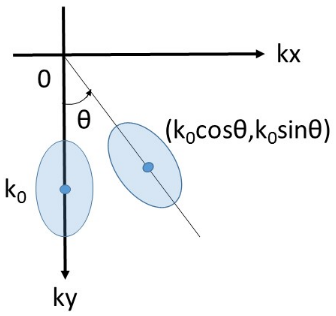
(2) Steering on monostatic synthetic aperture (SA)

Steering angles, transmission θt; reception θr,

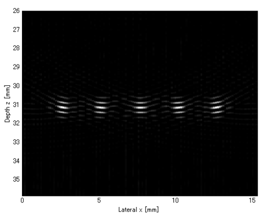
$$\exp(ik_x^t x) = \exp\{ik_0(\sin\theta_t + \sin\theta_r)x\}$$

$$\exp(ik_y^t y) = \exp\{ik_0(-2 + \cos\theta_t + \cos\theta_r)y\},$$

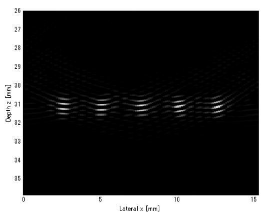
where k_0 is a wavenumber corresponding to US freq.



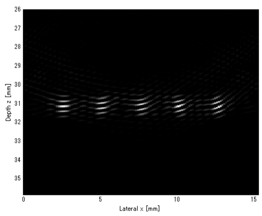
Schematic of spectra with steering angle θ



θt = θr = 0°



θt = θr = 5°(4.9°)



θt = θr = 10°(9.9°)

Combination of methods of (1) and (2)
[C. Sumi, IEEE Trans on UFFC (2008)]

Plane wave transmission with physical steering angle α,
steering angle θ via signal processing (software),
or both the steering angles α+θ;

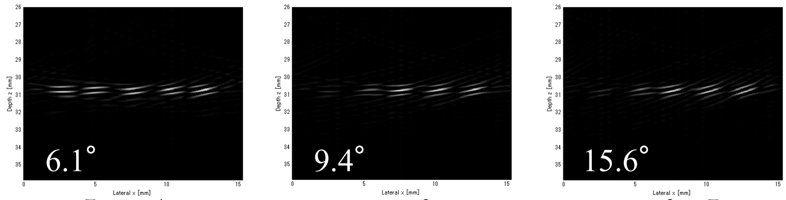
Reception dynamic focusing with steering angle Φ.

$$\exp(ik_x^t x) = \exp\{i(k\sin\theta + k_0\sin\phi)x\}$$

$$\exp(ik_y^t y) = \exp[i\{k\cos\theta + k_0(-1+\cos\phi)\}y],$$

where k_0 is a wavenumber corresponding to US freq.

e.g., Steering angle, 10°

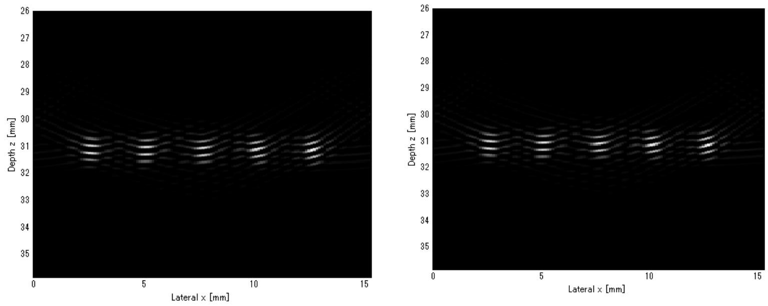


Φ solo | α+θ | α+θ+Φ

(3) Multistatic SA

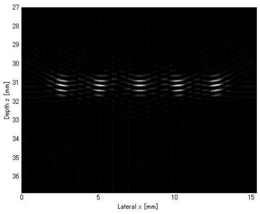
Performing monostatic SA with respect to respective echo data sets comprising of reception echo data of the same position with respect to the respective transmission position.

e.g., Reception steering angle, 10°

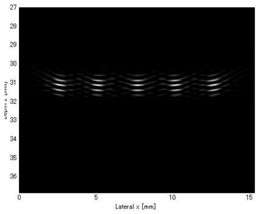


17 elements, 5.2° | 33 elements, 5.2°

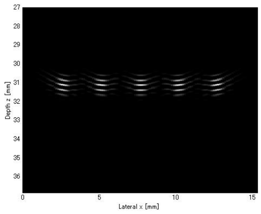
Effects of effective aperture width



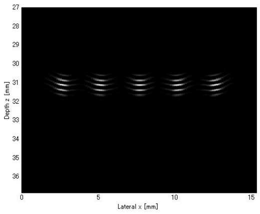
(a) 1 element



(b) 33 elements

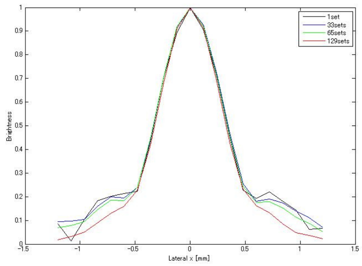


(c) 65 elements



(d) 129 elements

Suppression of sidelobes



Lateral beam profile

(4) Fixed focusing

Using method (1) for

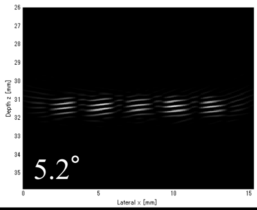
Method (4-1): all reception signals are compounded and processed.

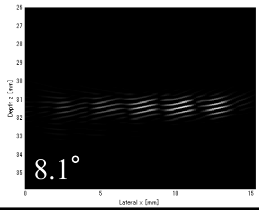
Method (4-2): low resolution images are generated for respective transmissions and compounded.

Method (4-3): similarly to multistatic SA, respective echo data sets are processed and compounded.

Method (4-1) the fastest of all.

e.g., Physical steering angles, 0° (left) and 10° (right), reception steering angle, 10° .

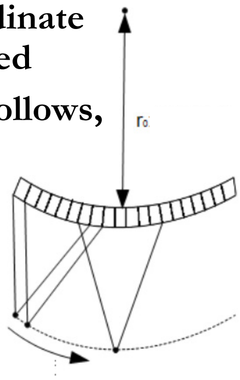




(5) Physical beamforming on polar system

Radial scan such as convex, sector, IVUS etc.

For echo data expressed using polar coordinate system, f(r,θ), angular spectra are calculated via 2D Fourier's transform and Jacobi as follows,



$$F(k_x, k_y) = \iint f(r,\theta) e^{-i(k_x x + k_y y)} dxdy$$

$$= \iint f(r,\theta) r e^{-i(k_x r\cos\theta + k_y r\sin\theta)} drd\theta,$$

where

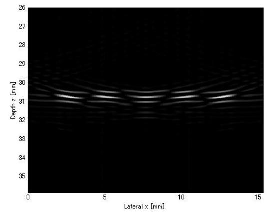
$$x = r\cos\theta, \quad y = r\sin\theta.$$

The beamforming can be carried out on the Cartesian coordinate system with no approximation.

cf. When achieving the beamforming on polar coordinate system (r,θ), angular spectra are calculated via 2D Fourier's transform on the orthogonal coordinate system (r,θ).

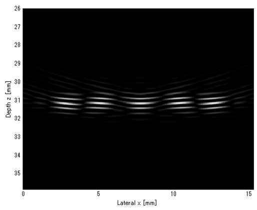
These approaches can also be used for an arbitrary coordinate system of physical beamforming or an arbitrary physical aperture geometry.

Convex type array probe:
radius, r_0=30mm;
Element width, 0.1mm;
Kerf, 0.02mm;
Element number 128.

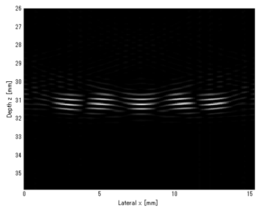


Cylindrical wave transmission and Method (1)

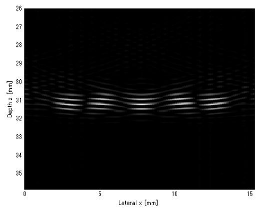
Fixed focusing, 30mm and Method (4)



Method (4-1)



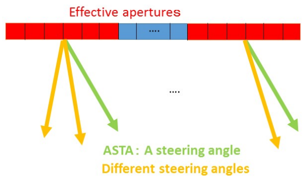
Method (4-2)



Method (4-3)

(6) Simultaneous signal processing for simultaneous transmissions of arbitrary beams or waves

Application of Method (1) or Method (8) for plane wave transmission to plural transmissions of focus beams, plane waves etc. High frame rate can be achieved by interrogating a large region simultaneously. Interferences become no problem.



1. Simultaneous transmissions in plural different directions or with multiple foci from each effective aperture
2. Simultaneous transmissions in the same direction (ASTA), different directions, or with the same focus or plural different foci from plural different effective aperture.

cf. Echo data received at different times, but the same phase of target can also be processed via coherent compounding similarly.

e.g., ASTA using focused beams

with physical transmission steering angle α,

steering angle β via signal processing (software),

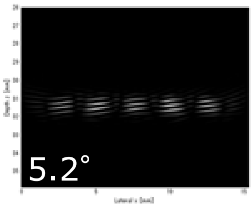
or both the steering angles α+β;

reception dynamic focusing with steering angle ɤ.

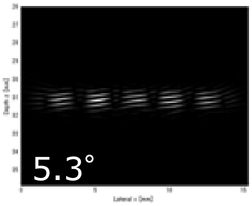
Table I. Generated steering angles of focused beams with respect to various sets of steering angles α, β and ɤ. Generated steering angles are evaluated using the 1st moments of spectra.

α	0°	0°	0°	10°	10°	10°	10°
β	0°	10°	0°	0°	10°	0°	10°
ɤ	0°	0°	10°	0°	0°	10°	10°
Method (1)	0.14°	5.2°	5.2°	3.4°	9.0°	8.1°	13.9°
Method (8)	0.14°	5.4°	5.3°	3.2°	9.2°	8.3°	13.9°

α=β=0°
ɤ=10°



Method (1)



Method (8)

For instance, as representative cases, i.e.,

where plural fixed focused beams or plural plane waves are transmitted,

of which physical transmission steering angles are different, and

(1) same software steering is performed

(2) plural different software steering is performed

and of which transmission steering angles are same,

(3) same software steering is performed, e.g., respective same physical steering angle, the same software steering can be added.

(4) plural different software steering is performed. These can be mixed.

The echo data to be applied to the same combination of software transmission and reception steering can be compounded and processed; whereas ones to be applied to different combination are respectively processed and compounded in a Fourier's domain.

For (1) and (3), software steering is performed one time on received echo data, i.e., compounded ones of respective which are generated with respect to the respective transmissions.

For (2), respective compounded echo data to be applied to the same software steering are processed one time, and compounded in a Fourier's domain.

For (4), Compounded echo data are applied to plural different software steering and compounded in a Fourier's domain.

(7) Virtual source or receiver

Reception beamforming with respect to

1) an arbitrary transmission beamforming such as fixed focusing, plane wave, circular wave, cylindrical wave, spherical wave, etc.

2) regardless the geometry of physical aperture

e.g., a circular wave is transmitted from a linear type probe,

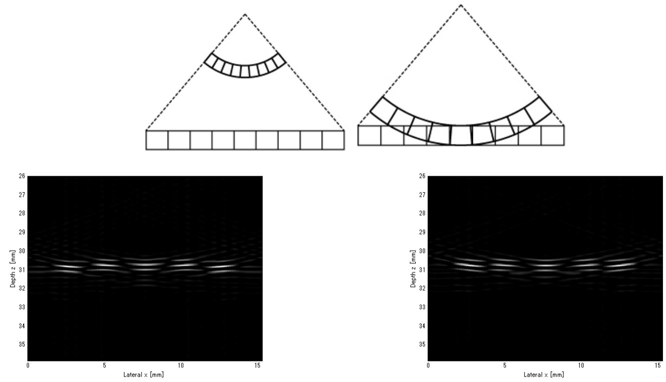
a large field of view (FOV),

a plane wave is transmitted from a convex type probe,

a constant depth is focused when using a convex-type probe, etc.

(a) A plane wave is generated at a depth using a convex type array probe, i.e., a virtual (wide) linear type array probe type.

Analogue processing: transmission or reception delay

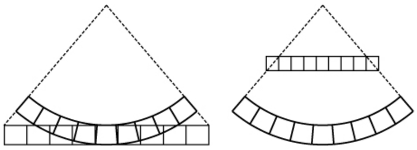


Transmission and reception

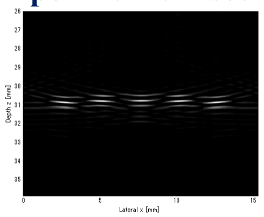
Reception solo (Jacobi processing on circular wave transmission)

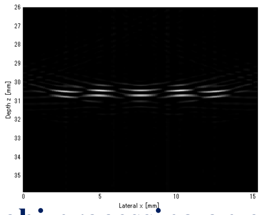
(b) A circular wave is transmitted from a linear type array probe, i.e., a virtual convex type array probe (a large FOV).

Analogue processing: transmission or reception delay



e.g., a point virtual source behind probe with 30 mm distance



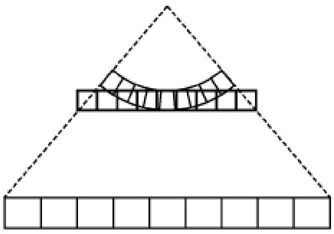


Processed on discrete Cartesian coordinate system (x,y) that expresses sampled, received echo data.

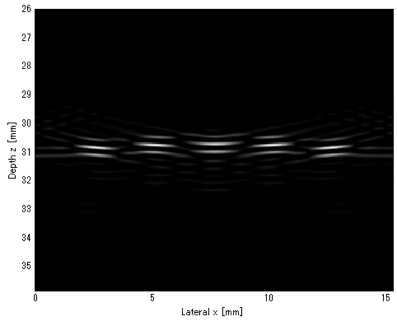
Jacobi processing on echo data re-expressed on a polar coordinate system (r,θ) via 1D Fourier's shifting of r(x,y).

(c) A wide plane wave is generated from a linear type array probe, i.e., a virtual wide linear type array (a large FOV).

Analogue processing: transmission or reception delay

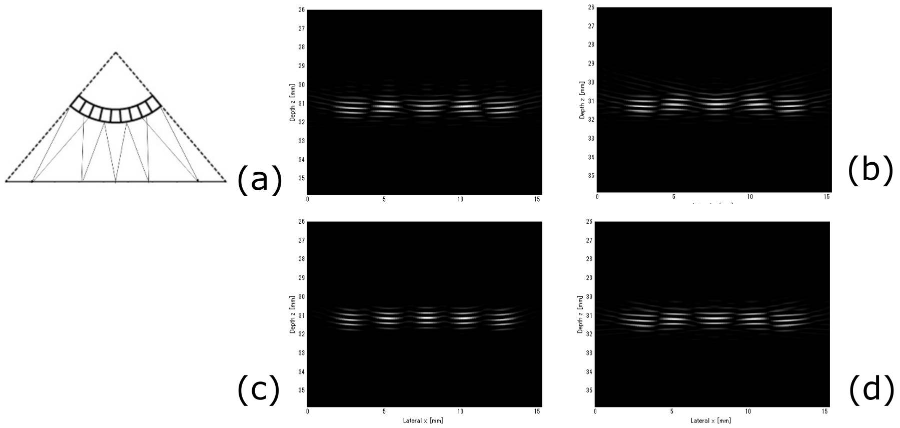


e.g., a point virtual source behind probe with 30 mm distance



Processed on discrete Cartesian coordinate system that expresses sampled echo data.

(d) Focusing at a constant depth using a convex type array probe



e.g., Focused at depth 30 mm: (a) transmission solo, (b) reception solo, (c) both. For (a) and (b), the respective reception and transmission focusing are performed on a constant distance 30 mm from an aperture of convex type probe.
(d) For (a), reception dynamic focusing is performed.

(8) Migration for Methods (1) to (7)

Conventional migration: monostatic SA with approximate wavenumber matching

We also perform no approximate wavenumber matching on migration.

(1) Plane wave transmission with steering

(2) Monostatic synthetic aperture (SA) with steering

(1)+(2) Combination of (1) and (2) [an effective beamforming, C. Sumi, IEEE Trans UFFC (2008)]

(3) Multistatic SA with steering

(4) Fixed focusing with steering

(5) Above-listed beamforming on an arbitrary coordinate system including a polar coordinate system

(6) Simultaneous signal processing for simultaneous transmissions

(7) Virtual source set behind physical aperture

With respect to Method (2),

Original migration,

$$\Phi(x,y,0)=\iint_{-\infty}^{+\infty}\frac{ck_y}{\sqrt{k_x^2+k_y^2}}\phi(k_x,0,f(k_y))\times\exp\{2\pi i(k_x x-k_y y)\}\,dk_x dk_y$$

No approximation,

$$\Phi(x,y,0)=\iint_{-\infty}^{+\infty}\phi(k_x,0,f)\times\exp\left\{2\pi i\left(k_x x-\sqrt{\left(\frac{f}{c}\right)^2-k_y^2}\,y\right)\right\}dk_x df$$

i.e., equivalent to Method (2).

With respect to Method (1),

transmission steering angle θt;
reception steering angle θr:

FFT on time t

Lateral matching $(\times e^{i(k\sin\theta_t+k_0\sin\theta_r)x})$

FFT on lateral direction x

Depth matching $(\times e^{\{ik_0(\cos\theta_r-1)\hat{y}\}})$ and calculation of angular spectra $\left(\times e^{\left\{2\pi i\left(-\sqrt{\left(\frac{f}{\hat{c}}\right)^2-(k_x-k_0\sin\theta_r)^2}\right)\hat{y}\right\}}\right)$

$\times\exp\left\{ik_y\hat{y}\frac{\sin\theta}{2-\cos\theta}\right\}$

IFFT on kx

where

$$\hat{y}=y(1+\cos\theta)^{\frac{3}{2}}/(1+\cos\theta+\sin^2\theta)$$

$$\hat{c}=c/\sqrt{1+\cos\theta+\sin^2\theta}\,.$$

Actually, approximations are performed when using Exploding Reflector Model [3].

With respect to Method (4),

in Method (1) realized by Method (8),

$\hat{k}\sin\theta_t$ is used instead of $k\sin\theta_t$,
where $\hat{k}$ is $2\pi f/\hat{c}$.

Conclusions

Various Fourier's beamforming methods with steering and no approximate wavenumber matching are reported. Through simulations, various beamforming (1) to (8) were performed.

Particularly in this report, Methods (6) and (7) are focused on. Simultaneous transmissions were performed and using various virtual sources, large FOVs were also obtained. Multiple focused beams, plane waves, a circular wave and virtual sources were used. These will further increase a frame rate.

Such fast beamforming will be effective for achieving measurements of rapid tissue motion, shear wave propagation and using 2D array type probe. The 3D versions can be realized in a straightforward manner (see patents etc).

These methods can also be used for electromagnetic waves such as lights, Terahertz etc.

Such applications will be performed in the future.

With reviews of our previous achievements:
All the methods are much more accurate than the corresponding approximate methods, i.e., no artifacts. An arbitrary reception beamforming can be performed with respect to received echo data of which sampled discrete coordinate system is different from that performed the reception beamforming. For instance, a transmission beamforming performed physically on a polar coordinate system (convex, radial scan, sector, IVUS etc) can be dealt with on a Cartesian coordinate system with no approximation. Reception beamforming can also be completed on the polar system with no approximation by similarly performing the angular spectra calculation on the orthogonal coordinate system, the polar system. These approaches can also be used for an arbitrary coordinate system of physical beamforming or an arbitrary physical aperture geometry.
The methods are much faster than Delay and Summation (DS) method, however, slower than the corresponding approximate methods. The difference in a calculation speed will be reported in detail elsewhere.